

Security Constrained Optimal Power Flow using Grey Wolf Optimization Technique

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Abstract: This article proposes grey wolf optimization (GWO) technique to solve Security constrained optimal power flow (SCOPF) problem. The proposed algorithm incorporates weighted penalty function for optimal adjustment of voltage and active power injection at generator bus along with ratio of tap changing transformer as control parameters to get the minimum total generation cost. The technique was tested on IEEE 30 bus system with quadratic cost function and totals 15 control parameters, both base case optimal power flow (OPF) and SCOPF solutions were obtained. In second case reactive power injection by shunt capacitors were also considered as control variables, the results were compared with other algorithms found in literature.

Keywords: Optimal power flow, Security constrained optimal power flow, Grey wolf optimization, IEEE 30 bus.

I. INTRODUCTION

Optimal power flow (OPF) is one of the most important aspects of power system operation and planning, control parameters like generator active power injection and voltages, transformer ratio and shunt/series capacitors and other control variables are adjusted to achieve optimal operating condition (such as minimum cost of fuel for thermal power plants) while making sure a set of operating constraints are not breached. But apart from economic, secure operation of power system is also a main concern, which leads to SCOPF- Security constrained optimal power flow, where the OPF problem is augmented to consider outage or contingency cases such as line outages, generation outages etc. to reach an operating point which is secure and optimal at the same time. The Security constrained OPF problem is highly non-linear and take huge computational time, as it uses iterative algorithms, convergence is not guaranteed. With increasing the size of network and more contingencies to be considered, this problem becomes more complex with computation time varying as the square of the number of nodes, and as the number of outages to be considered increases the search space becomes limited.

The OPF problem was first dealt by Dommel and Tinney [1] by Newton's method and then later B. Scott and O. Alsac [2] extended this problem by incorporating outage contingency constraints, which led to development of SCOPF. Later variety of other methods were applied to solve both OPF and SCOPF problems [3-7]. Though methods like quadratic programming, non-linear programming and interior point method were promising but seldom suffer from problem of poor convergence or difficulties in obtaining global solution. This conventional methods also required the objective function to be convex and differentiable, thus leads to approximated functions [8-9]. Application of Artificial intelligence based methods such as Genetic algorithm [10], Differential evolution [11], tabu search [12], Particle Swarm optimization [13] etc. to solve these kind of problem has increased because of better

performance than the methods discussed earlier. These methods can be easily implemented for parallel computing for faster results [14]. M.A. Abido [13] considered multiple objective function such as fuel cost minimization, voltage profile improvement and voltage stability enhancement for SCOPF problem. In recent times many other AI methods such as Artificial Bee colony [15], Bat algorithm, Imperialist Competitive algorithm etc. are suggested to solve both OPF and SCOPF problems. As No free lunch theory states that none of the algorithm can be considered best for all applications i.e. one might outperform other algorithm in certain applications, it has made search for new and more and more algorithms for application in power system. These heuristic approach can solve objective functions without worrying if it's differentiable or not or discrete or continuous, these are simple nature inspired algorithms easier to implement to any problems, Although there is no guarantee of exact optimal solution as in case of classical methods (which might also get sub-optimal solutions), but in power system analysis it is okay to have near optimal solution at right time rather than obtaining late results or no solution/no convergence at all.

In January 2014 S. Mirjalili, S. M. Mirjalili and A. Lewis [16] proposed a new meta-heuristic optimization technique inspired by the hunting behaviour and leadership hierarchy of Grey wolves (*Canis lupus*). Dipayan Guha et al [17] used this algorithm for Load frequency control in power system equipped with PI/PID controllers. In this paper grey wolf optimization technique is applied to solve both OPF and SCOPF problems with minimum cost of generation as objective function. Two cases are considered 1) 9 line outages are monitored with 15 control parameters and 2) Shunt capacitors are also considered as decision variables.

Rest of the paper is organized with introduction of SCOPF and problem formulation in Section II, Section III illustrates the grey wolf optimization technique and results

of programs on IEEE 30 bus test case are presented in Section IV, finally conclusions are drawn in section V.

II. SCOPF PROBLEM FORMULATION

SCOPF problem deals with adjusting controlling parameters of system to obtain optimal solution while considering security. But one thing that needs to be taken care of is, while obtaining optimal point the security of power system is not breached, i.e. even if there are any outages during operation the system can somehow manage the loss and remain in equilibrium even after outage. Few possible controllable parameters are:

- Generator MW outputs (P_{Gi})
- Generator voltages (V_{Gi} - of PV bus)
- Shunt reactors and capacitors – FACTS Devices
- Transformers with varying complex turn ratios
- Generating unit start-up/shut-down
- Line switching

SCOPF can be understood by first formulating optimal power flow problem, the objective function for minimum cost can be given by equation (1).

$$f = \sum K_i(P_{Gi}) \quad (1)$$

Where K_i is the cost function for generating P_{Gi} power at respective bus, in this paper a quadratic generation cost function is considered,

$$f(P_{Gi}) = aP_{Gi}^2 + bP_{Gi} + c \quad (2)$$

The objective of OPF problem is to minimize this function subject to, equality constrain i.e. power flow balance equation,

$$\bar{P}_k + j\bar{Q}_k = \bar{V}_k \sum_{m=1}^N \bar{Y}_{km}^* \bar{V}_m^* \quad (3)$$

The above equation can be re written as,

$$g(x, y) = \bar{P}_k + j\bar{Q}_k - \bar{V}_k \sum_{m=1}^N \bar{Y}_{km}^* \bar{V}_m^* = 0 \quad (4)$$

Where,

\bar{V}_k – Voltage at bus k

\bar{Y}_{km} – Element of bus admittance matrix

\bar{P}_k, \bar{Q}_k are real and reactive power injected at kth bus

Inequality constrains on control parameters such as, voltage at generator bus, Real power generation, and transformer tap ratio limits must be taken care of, along with functional constrains given by Voltage at PQ bus, Reactive power generation limits, Line overflow limits etc. Limits on control variables [u]

$$\begin{aligned} V_{g_{i(\min)}} < V_{g_i} < V_{g_{i(\max)}} a \\ P_{g_{i(\min)}} < P_{g_i} < P_{g_{i(\max)}} \quad (5) \\ a_{i(\min)} < a_i < a_{i(\max)} \end{aligned}$$

Functional constrains [x]

$$\begin{aligned} V_{PQi(\min)} < V_{PQi} < V_{PQi(\max)} \\ Q_{g_{i(\min)}} < Q_{g_i} < Q_{g_{i(\max)}} \quad (6) \\ S_{l(\min)} < S_l < S_{l(\max)} \end{aligned}$$

Therefore, above discussed OPF problem looks like eq. 7

$$\begin{aligned} \min_{[u]} f(x, u) \\ \text{subject to equality constrain} \\ [g(x, u, p)] = 0 \\ \text{and inequality constrains} \quad (7) \\ [u^{\min}] \leq [u] \leq [u^{\max}] \\ [x^{\min}] \leq [x] \leq [x^{\max}] \\ \text{i.e. } h(x, y) \leq 0 \end{aligned}$$

Where,

[x] – Matrix of state variables

[u] – Controllable variables

[p] – Fixed variables

$$[x] = \begin{bmatrix} V \text{ and } \delta \text{ on each PQ bus} \\ \delta \text{ on each PV bus} \end{bmatrix} \quad [y] = \begin{bmatrix} V_i \\ \delta_i \\ P_i \\ Q_i \\ P_i \\ V_i \end{bmatrix} \begin{array}{l} \text{Slack bus} \\ \text{for PQ bus} \\ \text{for PV bus} \end{array}$$

This OPF problem can be now further extended to SCOPF by implementing following strategy:

- i. Solve the optimal base-case load flow
- ii. Monitor outage-contingency
- iii. If insecure case is found, impose constrain corresponding to that.
- iv. Again go to (ii.) until final optimal is obtained.

New augmented objective function after imposing security constrains now becomes,

$$F = f(x^o, u) + \sum_{k=0}^s \sum_j w_j^k(x^k, u) \quad (8)$$

Where, w_j^k is penalty introduced for each violated constrain j in each outage case k, s is number of outage case detected insecure. Now this augmented function can be solved by any optimization technique.

III. GREY WOLF OPTIMIZATION

It can be observed that SCOPF problem is highly complex problem. With increase in the size of network, the problem becomes more complex and search space more limited, to solve this kind of problem Artificial Intelligent technique for optimization is found to be recent trend.

Grey wolf optimization method is a meta-heurist technique inspired by the hunting behaviour and leadership hierarchy of Grey wolves [16]. Grey wolves prefer to live in a pack of size 5 to 12 and have a very dominant social hierarchy. The leaders are called alphas. They are called decision makers as rest of the wolves follow his/her orders. Beta are the subordinate wolves that come on second level of the hierarchy, they help alpha in decision-making or other pack activities. They are the best substitute for alphas in case it dies or is old enough. It plays the role of manger of

the pack and advisor to the alpha. The lowest ranking grey wolf is omega. They are at the bottom of the hierarchy they are allowed to eat at last and plays role of scapegoat. Omegas have to report to all other wolves. If a wolf is not an alpha, beta, or omega, then it is called delta. They have to submit to alphas and betas, but they dominates omega. Scouts, hunters, elders, sentinels, and caretakers belongs to this category. Apart from the social hierarchy of grey wolves, they also depict another interesting behaviour of group hunting.

The main phases of grey wolf hunting are: (see Fig. 1)

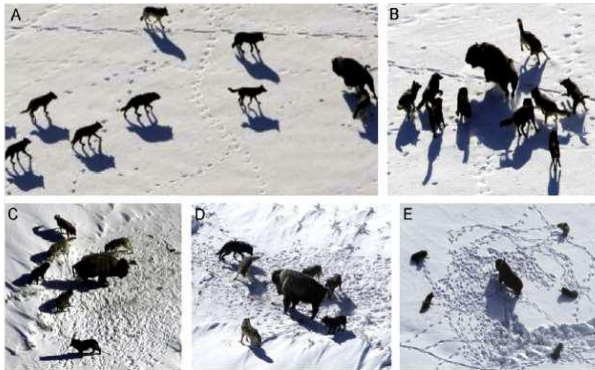


Fig. 1 hunting behaviour of grey wolves [16]

- Tracking, chasing, and approaching the prey (A).
- Pursuing, encircling, and harassing the prey until it stops moving (B-D).
- Attack towards the prey (E).

A. Mathematical model

The above social behaviour of Grey wolves is mathematically modelled and then optimization algorithm is developed.

1. Social hierarchy: The fittest solution is considered as the alpha (α), the second and third best solutions are beta (β) and delta (δ) respectively. The rest of the possible solutions are assumed to be omega (ω). Further hunting or optimization is guided by positions of α , β and δ and ω wolves follow these three wolves.

2. Encircling prey: Grey wolves encircle a prey during the hunt. Following equations are proposed to mathematically model encircling behaviour of grey wolves:

$$\begin{aligned} \vec{D} &= \left| \vec{C} \cdot \vec{X}_p(t) - \vec{X}(t) \right| \\ \vec{X}(t+1) &= \vec{X}_p(t) - \vec{A} \cdot \vec{D} \end{aligned} \quad (9)$$

Where,

t - Current iteration

X_p - Position vector of the prey

X - Position vector of a grey wolf.

Vectors A and C are coefficient vectors calculated as follows

$$\begin{aligned} \vec{A} &= 2a \cdot \vec{r}_1 - a \\ \vec{C} &= 2 \cdot \vec{r}_2 \end{aligned} \quad (10)$$

Where, r_1, r_2 are random vectors in $[0, 1]$ and a is linearly decreased from 2 to 0 over the course of iterations.

3. Hunting: Grey wolves recognizes the location of prey and encircle them. Alpha then guides other wolves for hunt. The beta and delta might also help in hunting. But in any optimization problems we don't know the exact solution or the location of prey and thus we take help of alpha (best known solution) beta and delta to estimate the position of prey and guide other wolves towards the same. Following equations are used for updating the position of search agents based on location of α , β and δ .

$$\begin{aligned} \vec{D}_\alpha &= \left| \vec{C}_1 \cdot \vec{X}_\alpha - \vec{X} \right|, \quad \vec{X}_1 = \vec{X}_\alpha - \vec{A}_1 \cdot (\vec{D}_\alpha) \\ \vec{D}_\beta &= \left| \vec{C}_1 \cdot \vec{X}_\beta - \vec{X} \right|, \quad \vec{X}_2 = \vec{X}_\beta - \vec{A}_2 \cdot (\vec{D}_\beta) \\ \vec{D}_\delta &= \left| \vec{C}_1 \cdot \vec{X}_\delta - \vec{X} \right|, \quad \vec{X}_3 = \vec{X}_\delta - \vec{A}_3 \cdot (\vec{D}_\delta) \end{aligned} \quad (11)$$

$$\vec{X}_p(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3}$$

4. Attacking prey (exploitation): In order to mathematically model attacking behaviour we need to decrease the value of A , therefore the value of 'a' is decreased from $[2 \text{ to } 0]$, as the value of A will always be between $[-a, a]$ if $|A| < 1$ then eq. 9 will force wolves to move towards the prey.

5. Search for prey (exploration): Grey wolves updates their position according to position of the alpha, beta, and delta. They diverge from each other or explore the search space to search for prey and converge or exploit to attack prey.

- If $|A| < 1$ == Attacking prey – Exploitation
- If $|A| > 1$ == Searching for prey – Exploration

Another parameter that favours exploration is vector C , its values is between $[0, 2]$ and it can be considered as a hurdle for wolf to reach towards prey, if $C > 1$ it emphasise or if $C < 1$ it deemphasise the effect of distance D .

B. Implementing SCOPF using GWO

Figure 2 shows the flow chart to solve SCOPF problem, a search agent would look like

$$\left[\begin{array}{cccc} V_{g_s} & V_{g_1} & \dots & V_{g_{nPV}} \end{array} \right] \quad \left[\begin{array}{cccc} P_{g_1} & P_{g_2} & \dots & P_{g_{nPV}} \end{array} \right] \quad \left[\begin{array}{cccc} a_1 & a_2 & \dots & a_{nTAP} \end{array} \right]$$

Where,

a_i is transformer tap setting

$nTAP$ - no. of transformers

nPV - no. of generator bus (not including Slack bus)

Therefore, size of total population would be,

$$[(pop + 3) \times (2 \times nPV + nTAP + 1)]$$

Where, pop = number of search agents

Generation at slack bus is not considered as a decision variable and is computed from power flow solution.

Initially network data is fed to the programme such as, bus data, line data and generation cost coefficients, and then random positions for search agents are generated based on number of decision making variables. Power flow solution is obtained for each search agent and fitness is calculated based on objective function, penalty is added in case there is any constrains are breached. Based on this fitness, positions of all the search agents are updated using GWO programme, it is expected that the fitness obtained in next iteration would be better than previous.

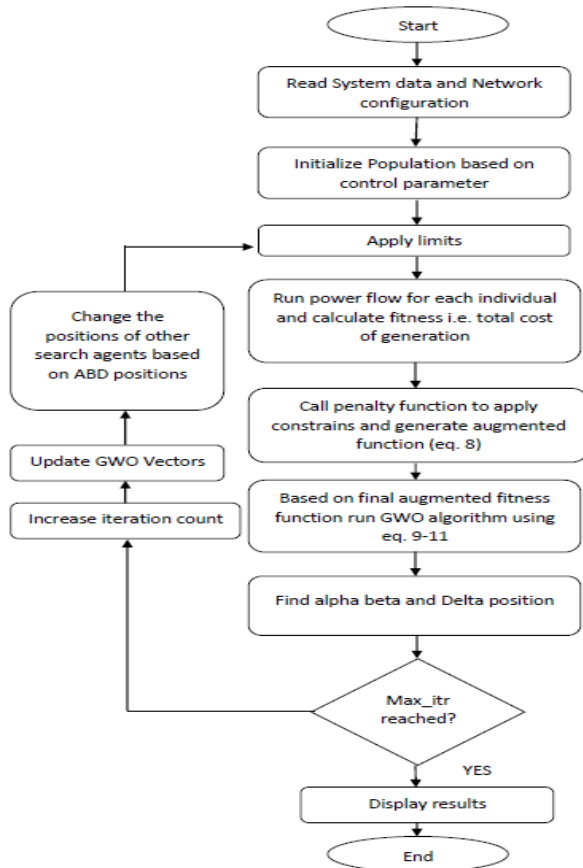


Fig 2. Flow chart for SCOPF using GWO

IV. SCOPF ON IEEE 30 BUS TEST CASE

IEEE 30 bus test case [2] was considered to demonstrate the performance of GWO algorithm, MATLAB (2012) code was developed to obtain the optimal value of control parameters and MATPOWER 5.0 software package was used to obtain power flow solution. The objective function considered here is the total active power generation cost, the summary of the test case is given in table I, and while detail data such as line data and bus generation are given in [2]. The program is implemented on a personal computer with Intel i3 quad core 2.4 GHz processor and 4 GB total memory.

TABLE I SUMMARY OF TEST CASE

No. of Busses	30
Slack bus number	1
Generators	6
Loads	21
Shunts	2
Transformer	4
Branches	41

Two different cases are considered to analysis the performance of the proposed algorithm.

- i. Optimal power flow and SCOPF program on IEEE 30 bus test case with quadratic cost function and total 15 control variables.
- ii. Base case OPF and SCOPF by adding shunt capacitors as control variables i.e. total 24 decision variable.

A. Case 1

As depicted in flowchart SCOPF problem was divided in to sub problems, i) base case OPF on test system was solved. ii) The results obtained from OPF solution were taken as input (or rather initial conditions) for obtaining solution for SCOPF problem. Voltage limits for generator bus were 0.95 to 1.1 p.u. while for load bus were 0.95 to 1.05 p.u transformer tap ratio were allowed within $\pm 1\%$. For SCOPF case total 9 line outage were taken which are shown in table II.

TABLE II DETAILS FOR CASE 1

Control parameters	No.		Line outage
Generator bus voltage (V_i)	6	$V_{g_1}, V_{g_2}, V_{g_5}, V_{g_8}, V_{g_{11}}, V_{g_{13}}$	1
			2
Generator active power injection (P_{gi})	5	$P_{g_2}, P_{g_5}, P_{g_8}, P_{g_{11}}, P_{g_{13}}$	4
			5
			7
Transformer tap ratio (a_i)	4	$a_{11}, a_{12}, a_{15}, a_{36}$	33
			35
Total	15		37
			38

GWO programme to solve both the problem was set to run for 200 iterations and with 25 search agents for base case OPF and 100 iterations and 20 search agents for SCOPF problem, total 20 trials were taken, the summary of results are given in table IVs. When we compare this results as found in literature we can see that performance of Grey wolf optimization technique is satisfactory (table IV).

TABLE III DETAILS FOR CASE 2

Control parameters	No.		Line outage
Generator bus voltage (V_i)	6	$V_{g_1}, V_{g_2}, V_{g_5}, V_{g_8}, V_{g_{11}}, V_{g_{13}}$	41
Transformer tap ratio (a_i)			
	Shunt capacitors (Q_i)		
Total		24	

B. Case 2

In this case few more control parameters i.e. reactive power injection through shunt capacitors, and only one line outage contingences are taken into consideration [19]. Again 20 trials of GWO for 200 iterations with 20 search agents were executed and the results for these trials are shown in table V. Reactive power injection limit for each shunt was considered between 0 and 0.05 p.u. i.e. (0 to 5 MVar). Results when compared to that found in literature, it is observed the GWO algorithm performs better than most of the algorithms; rest of the data is similar to that of case 1. By introducing shunt capacitors the total cost of generation of active power obtained in results is reduced marginally.

TABLE IIIV COMPARISON WITH RESULTS FOUND IN LITERATURE – CASE 1

Case 1	Algorithm								
	Gradient [2]		EP [18]	IEP [11]	TS [12]	pSADE [14]		GWO	
	OPF	SCOPF	OPF	OPF	OPF	OPF	SCOPF	OPF	SCOPF
Best cost	802.4	813.74	802.62	802.465	802.29	802.405	826.978	802.7480	804.6861
Average cost	-	-	803.5	802.521	-	802.405	826.978	803.7199	815.8672
Worst cost	-	-	805.6	802.581	-	802.405	826.978	806.6341	810.2796
STD	-	-	-	0.039	-	0	0	0.86	11.72
Computing time(s)	-	-	51.4	99.013	-	17.29	157.401	32.2	120

TABLE V COMPARISON WITH RESULTS FOUND IN LITERATURE – CASE 2

Case 2	Algorithm					
	Gradient[20]	PSO[13]	IGA [19]		GWO	
	OPF	OPF	OPF	SCOPF	OPF	SCOPF
Best cost	804.853	800.41	800.805	812.33	800.0199	804.5961
Average cost	-	-	-	-	800.6328	825.9834
Worst cost	-	-	-	-	801.5110	903.7648
STD	-	-	-	-	0.47	21.85
Computing time(s)	4.324	-	5.25 (min)	6.91 (min)	34.6	45

The decision variables for the best solution obtained in both cases are shown in table VI. Rated line data for each case is considered to be their maximum line flow limits. Initial penalty weights for SCOPF are adjusted as [5 1 1] for V_{pqi} , Q_{si} and S_{ii} respectively and [20 0.02 0.1] for OPF.

TABLE VI CONTROL PARAMETERS FOR EACH CASE

		Case 1		Case 2	
		OPF	SCOPF	OPF	SCOPF
V	1	1.05	1.049	1.1	1.05
V	2	1.039	1.029	1.087	1.035
V	5	1.01	0.994	1.055	1.008
V	8	1.021	1	1.067	1.015
V	11	1.1	1.062	1.1	1.091
V	13	1.093	1.1	1.087	1.079
P	1	175.49	175.18	174.91	167.98
P	2	47.21	48.77	51.05	50.72
P	5	21.27	18.01	21.57	19.61
P	8	22.77	19.43	18.24	20.28
P	11	11.54	17.06	13.45	18.92
P	13	14.52	14.63	12.93	14.92
'a'	11	0.9572	0.95	1.0062	0.9859
'a'	12	0.98	0.9642	0.977	1.0065
'a'	15	0.9798	1.05	1.0361	1.0342
'a'	36	0.958	0.95	0.974	0.95
Q	10	-	-	1.914	1.629
Q	12	-	-	4.016	1.758
Q	15	-	-	4.313	2.779
Q	17	-	-	0.787	2.2
Q	20	-	-	3.052	2.49
Q	21	-	-	0.055	2.084
Q	23	-	-	1.545	0.8
Q	24	-	-	1.042	1.815
Q	29	-	-	2.318	2.94
Cost		802.748	804.6861	800.02	804.596

V. CONCLUSION

SCOPF problem are highly complex, non-linear and requires lot of computational time. AI technique can be used as it has more flexibility and adaptability towards the problem. Using Grey wolf optimization technique we are able to get promising result and can be easily implemented. Penalty function based approach was used for converting constrained optimization function to unconstrained one. We can see that the cost of generation of SCOPF problem is more than that of OPF, this extra cost is a trade-off between the Security and economy, the constrained imposed due to security makes the search space narrower and resulting in smaller feasible region. Implementation of GWO on IEEE 30 bus test case gave results similar (or better in some case) to that found in literature.

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