

Optimal Unit Commitment

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Abstract: In this paper, the Unit Commitment (UC) problem has been solved using Dynamic Programming (DP). The test results of conventional Dynamic Programming and Modified Dynamic Programming are compared. The commitment is such that the total cost is minimal, which is non-linear function. The total cost includes both the production cost and the costs associated with start-up and shutdown of units. DP is an optimization technique which gives the optimal solution.

Keywords: Unit Commitment, Dynamic Programming (DP), Modified Dynamic Programming, Non-linear.

I. INTRODUCTION

Unit Commitment (UC) problem of power systems dates back to the 1940's [6]. The task of Unit Commitment (UC) involves scheduling the on/off status, as well as the real power outputs, of thermal units for use in meeting forecasted demand over a future short-term (24–168 hour) horizon [8]. The objective is to minimize total cost while observing a large set of operating constraints. The total cost consists of production cost, Start-up cost and Shut down cost. The objective is to minimize total cost while observing a large set of operating constraints. The total cost consists of production cost, Start-up cost and Shut down cost. The Unit Commitment Problem (UCP) mathematical optimization problem with both integer and continuous variables. The exact solution to the problem can be obtained by complete enumeration, which cannot be applied to realistic power systems due to its excessive computation time requirements. Further, UC problem must take into account a large number of practical constraints. These constraints are:

- System power balance (demand plus loss and export).
- Unit initial conditions.
- System reserve requirement
- Unit minimum up-time.
- Unit minimum down-time
- Unit generation capability (upper/lower) limits.
- Unit start-up ramps
- Unit shut-down ramps
- Unit dual or alternate fuel usage
- Unit dual or alternate fuel usage

The first two called system or coupling constraints. Other concern individual units and are called local constraints. Plant crew constraints can also be classified along with local constraints but they involve all units in a plant. Different aspects of UCP are: (1) Multi-area constrained unit commitment: Many utilities and power pools have limits on power flow between different areas/regions over tie lines. Each area/region have its own pattern of load variation and generation characteristics. They also have separate spinning reserve constraints. The system constraint is modified to take into account the interchange

schedules and the tie-line limitations. (2) Multi-objective unit commitment: In contrast to existing UC solution, this method treats economy, security and reliability as competing objectives for optimal UC solution. A compromised solution is preferred as most of these objectives are conflicting and improvement of one objective may degrade the performance of another. Artificial Intelligent (AI) techniques can be used to solve these type of UC problem. There are many UC methods such as lagrangian relaxation [2], priority list method, the dynamic programming which is introduced in this paper. All of these methods have their advantages and disadvantages. Lagrangian relaxation algorithms are efficient in speeding up the solving of optimisation problems [2]. The constraints are divided into two types, the equalities and inequalities. With Langrangian Relaxation (LR) methods, due to the duality gap, there is no guarantee of the optimality of the solutions. In priority list method (PL), the units are committed in ascending order of the unit Average Full Load Cost so that the most economic base load units are committed first and the peaking units last in order to meet the load demand. PL methods are very fast but they are highly heuristic and give schedules with relatively high production costs. The basis for Dynamic Programming (DP) is the theory of optimality proposed by Bellman in 1957[11]. By using dynamic programming for unit commitment, we can get optimal solutions. But solution of large scale UC problems using conventional DP is time consuming because it involves complete enumeration of units instead it gives the best optimal solution.

II. PROBLEM FORMULATION

The objective function of the thermal UC problem can be composed of the fuel and start-up costs for the generating units and can be express as:

$$\text{Min } F_T = \sum_{t=1}^H \sum_{i=1}^N \{FC_{i(t)}(P_{i(t)})U_{i(t)} + CU_{i(t)}U_{i(t)}[1 - U_{i(t-1)} + C_{DOWN}N_{i(t-1)}U_{i(t)}]\}$$

Where,

F_T - The total operating cost over the scheduled horizon,
 $FC_{i(t)}(P_{i(t)})$ - The fuel cost function for unit i ,
 $P_{i(t)}$ - Generation output of unit i at hour t ,
 CUP_i - Start-up cost of unit i ,
 $CDOWN_i$ - Shut-down cost for unit i ,
 $U_{i(t)}$ - The on/off status of unit i at hour t , $U_{i(t)}=0$ when off and $U_{i(t)}=1$ when on,
 $U_{i(t-1)}$ -The on/off status of unit i at hour $t-1$, $U_{i(t-1)}=0$, when off and $U_{i(t-1)}=1$ when on,
 U - The decision matrix of the $U_{i(t)}$ variable for $i = 1, 2, \dots, N$,
 N - The number of thermal generating units,
 H - The number of hours in the study period.
 The fuel cost function of the thermal unit $FC_{i(t)}(P_{i(t)})$ is expressed as a second-order polynomial:

$$FC_{i(t)}(P_{i(t)}) = A_i + B_i \times P_{i(t)} + C_i \times P_{i(t)}^2$$
 for $i = 1, \dots, N$
 $t = 1, \dots, H$
 Where A_i, B_i, C_i are constants.

Constraints:

1. Generation Constraints:

$$\sum_{i=1}^N PS_{i(t)} \times U_{i(t)} = PD_{i(t)}$$

Where $PD_{i(t)}$ is the system load demand at hour t ,
 $PS_{i(t)}$ is the power out of unit i at hour t .

2. Unit Generation Limitations:

$$PS_{imin} \leq PS_{i(t)} \leq PS_{imax}$$

Where, PS_{imin}, PS_{imax} , are the minimum and maximum power output of unit i respectively.

3. Minimum Up-Time:

Unit i must be kept on if it is up for less than the minimum-up-time, i.e.

$$T_{oni} \geq MU_i$$

T_{oni} is the up-time of unit i ,

MU_i is the minimum up-time of unit i .

4. Minimum Down-Time:

Unit i must be kept off if it is down for less than the minimum down time, i.e.

$$T_{offi} \geq MD_i$$

T_{offi} is the down-time of unit i ,

MD_i is the minimum down-time of unit i .

There are mainly three types of generator starting method.

- 1) Cold start-up (constant)
- 2) Cold / hot start-up
- 3) Exponential start-up

In this paper, second method – cold / hot start-up is considered. If generator is down for the time more than minimum down time plus cold starthour, cold start-up cost is considered. If generator is down for the time less than minimum down time plus cold start hour, hot start-up cost is considered.

III. DYNAMIC PROGRAMMING APPROACH

DP methods decompose the UC problem in time. Starting at the first hour of the scheduling, commitment of units progresses one hour at a time, and combinations of units are stored for each hour. Also the time of operation that is how much time the generator has remained off or on is also saved to satisfy the minimum uptime and minimum down time limit constraints. The method of starting can be cold start or hot start, according to that transition cost is calculated. This is the forward path of the DP method. Within a time period the combinations of units are known as the states. The main problem of the DP methods [5] is storing all possible unit combinations ($2^N - 1$, N : number of units) at every hour is very difficult even for moderate size systems. Assume that there are 4 units which can supply the 24 hour load. So, the total maximum path to satisfy the 24 hour load curve is calculated by:

$$\text{Total Paths} = (2^4 - 1)^{24}$$

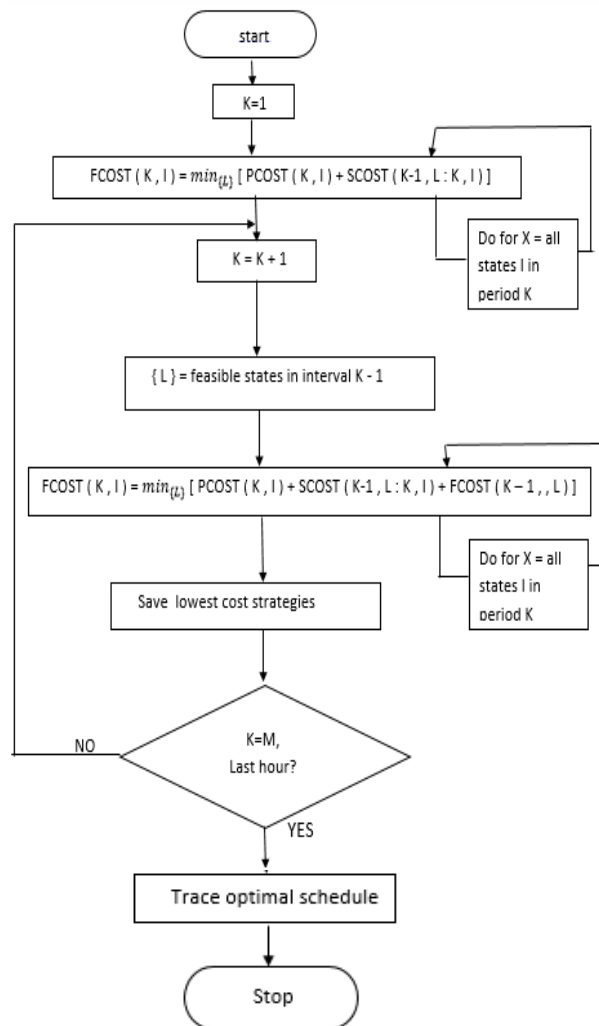


Fig 1. Flowchart for the Dynamic Programming

Fig. 1 shows flowchart for the DP. Although the solution of DP is correct and has the optimal value, it takes a lot of memory and spend much time in getting an optimal solution.

Because of this disadvantage, heuristic techniques are used to restrict the number of combinations to be searched and the number of strategies to be saved at every hour. This heuristics produce suboptimal solutions and in certain cases may require the relaxation of some of the constraints in order to produce a solution. Sometimes modified Dynamic Programming also used. In this method, only some of the feasible states are taken into account. That may increase the cumulative cost but memory needed is less.

The recursive algorithm to compute minimum cost in hour K with I combination is:

$$F_{cost}(K, I) = \min_{\{L\}} [P_{cost}(K, I) + S_{cost}(K-1, L; K, I) + F_{cost}(K-1, L)]$$

Where,

$F_{cost}(K, I)$ = Least total cost to arrive at state (K , I)

$P_{cost}(K, I)$ = Production cost for state (K , I)

$S_{cost}(K-1, L; K, I)$ = Transition cost from state (K-1 , L) to state (K , I)

$F_{cost}(K-1, L)$ = Least total cost at state (K , I).

Modified dynamic programming does not save all the paths in order to get the optimal solution. At K periods, we consider all the feasible states X which could be satisfied by demanding from N paths at the K-1 period as shown in Fig. 1 and 2. Continuously, we find the lowest new N paths and thereby save memory and time. Similarly, we iterate it until the last period.

Here,

N: Number of Saving Paths

X: Feasible States Satisfied Load Curve

M: Total Period

K: Current Period

All the other parameters are same as dynamic programming.

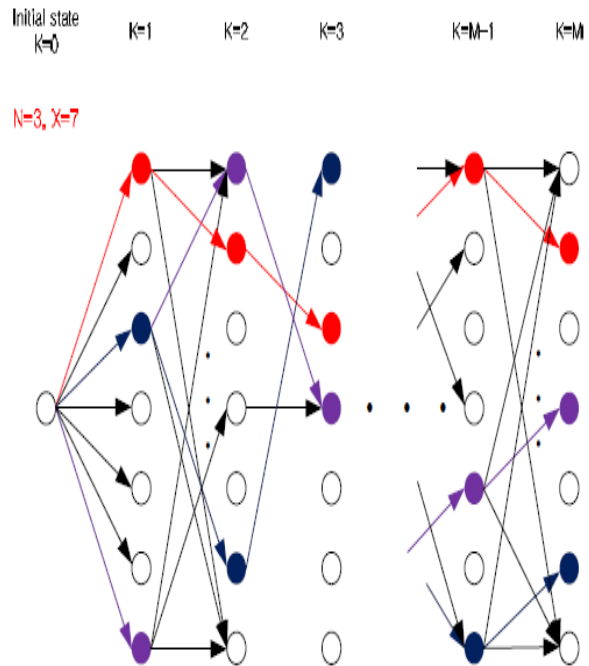


Fig 2. The Modified dynamic programming path saving

IV. CASE STUDY

The results for DP are presented and tested on 5 unit base system with a 24-hour time horizon. The program was written in MATLAB. The input data for 5 unit system and load demands for 24 hours are shown in Tables I and Table II respectively.

TABLE I Data for 5-Unit System

	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
Pmin (Mw)	150	20	20	20	55
Pmax (Mw)	455	130	130	80	55
a (\$/h)	1000	700	680	370	660
b (\$/Mwh)	16.19	16.6	16.5	22.26	25.92
c (\$/Mw^2h)	0.00048	0.002	0.00211	0.0072	0.00413
Min. up time (h)	8	5	5	3	1
Min. down time (h)	8	5	5	3	1
Hot start cost (\$)	4500	550	560	170	30
Cold start cost (\$)	9000	1100	1120	340	60
Cold start hours (h)	5	4	4	2	0
Initial status (h)	8	-5	-5	-3	-1

TABLE II Load demand for 24-hours

Hour (h)	Demand (Mw)	Hour (h)	Demand (Mw)	Hour (h)	Demand (Mw)
1	330	9	620	17	650
2	450	10	650	18	670
3	480	11	680	19	790
4	360	12	630	20	750
5	520	13	810	21	770
6	590	14	820	22	610
7	730	15	750	23	520
8	780	16	800	24	360

TABLE III Power generated by each unit using DP

Hours	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
1	330	0	0	0	0
2	450	0	0	0	0
3	455	0	0	0	25
4	360	0	0	0	0
5	455	0	65	0	0
6	455	0	115	20	0
7	455	125	130	20	0
8	455	130	130	65	0
9	455	62	83	20	0
10	455	78	97	20	0
11	455	93	112	20	0
12	455	67	88	20	0
13	455	130	130	80	15
14	455	130	130	80	25
15	455	130	130	35	0
16	455	130	130	75	10
17	455	78	97	20	0
18	455	88	107	20	0
19	455	130	130	75	0
20	455	130	130	35	0
21	455	130	130	55	0
22	455	0	130	25	0
23	455	0	65	0	0
24	360	0	0	0	0

Table III shows power generated by each generating unit, which is calculated using Dynamic Programming. It is observed that total cost at the end of the day obtained by DP is 312980\$. The problem with DP is as no. of unit increases the total

path to be examined also increases which require more memory to store data as well as more time.

Figure 3 shows how time require to solve UC problem using DP increases as no. of units increase

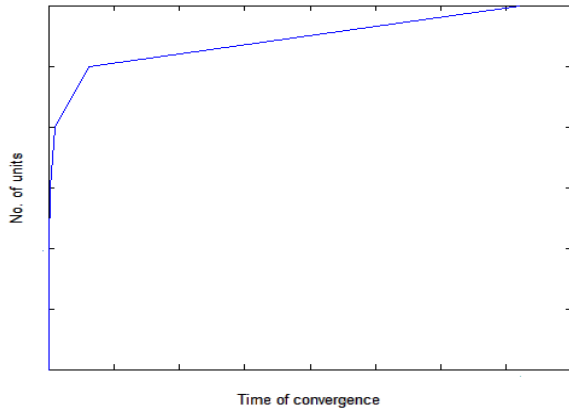


Figure 3. Increment in time of convergence with increase in No. of units

TABLE IV Power generated by each unit using Modified DP

Hours	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
1	330	0	0	0	0
2	450	0	0	0	0
3	455	25	0	0	0
4	340	20	0	0	0
5	455	45	0	20	0
6	455	47	68	20	0
7	455	125	130	20	0
8	455	130	130	65	0
9	455	71	94	0	0
10	455	88	107	0	0
11	455	102	123	0	0
12	455	79	98	0	0
13	455	130	130	80	15
14	455	130	130	80	25
15	455	130	130	25	10
16	455	130	130	75	10
17	455	78	97	20	0
18	455	88	107	20	0
19	455	130	130	75	0
20	455	130	130	35	0
21	455	130	130	55	0
22	455	65	90	0	0
23	455	0	65	0	0
24	360	0	0	0	0

It is observed that total cost at the end of the day obtained by modified dynamic DP is 334720\$. Which is more than cost obtained from DP, but using modified dynamic method memory required will be less

V. CONCLUSION

Using DP method, we can get optimal solution but it requires more memory to save data and more time to solve problem. It also has major difficulty in treating time-dependent constraints such as unit minimum up and down times, time dependent start-up costs, start-up ramps etc. This difficulty may lead to suboptimal solutions or failure to provide solution even in the case of complete state enumeration. The modified DP requires less memory and

time by saving the paths but it gives total cost higher than DP method. To solve these difficulties Artificial Intelligence techniques can be used.

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