Application of Bisection Method

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ABSTRACT: The method used in this project is called a bisection method. It is an approximation method that finds the roots of an equation by repeatedly dividing an interval.

INTRODUCTION

The bisection method is a prominent mathematical and numerical root-finding method. This approach can be used to discover the root of any polynomial equation $f(x) = 0$, as long as the roots are contained inside the range.

The interval [a, b] is continuous, and f(x) is continuous. This is a closed bracket approach that requires two initial guesses. The Convergence is linear, and overall accuracy is good. When compared to in comparison to other rooting procedures, the bisection method is thought to be the most effective. Because of its moderate and consistent rate of convergence, it is rather slow.

THEORY

The Bisection method is one of the most straightforward and dependable iterative approaches for solving nonlinear equations. This method, sometimes known as binary chopping or the half-interval method, is based on binary chopping. Since f(x) in the interval axb is real and continuous, and f(a) and f(b) have the same sign, i.e., f(a) f(b)0. Between a and b, there is at least one real root. It's possible. In the interval, there are multiple roots. Let's say x1=a and x2=b. Let us also consider designate a third point x as the halfway point between a and b is

$(x1+x2)/2 = x0$

There are currently three conditions in place:

1. We have a root at x0 if f(x0) =0.

2. If f(x0) f(x1)0, then between x0 and x1 there is a root.

3. If f(x1) f(x2)0, a root exists between x1 and x2.

As a result, we can test the function's sign at the halfway. ascertain which portion of the interval comprises the root, this is demonstrated in the following example. Below is a diagram. It demonstrates that, because f(x0) and f(x2) have the same sign, between x0 and x2, there is a root. This subinterval can be subdivided further to find a new subinterval containing the root, divide it in half. This technique can be repeated until the root interval is as small as we want.

The Intermediate Value Theorem is used in the bisection method. "If a continuous function f(x) exists in the interval [a, b], and c is any integer between f(a) and f(b), then there exists at least one number x in that interval such that f(x) = c," says the theorem.

The following is a diagram of the intermediate value theorem:

Fig: Illustration of bisection method
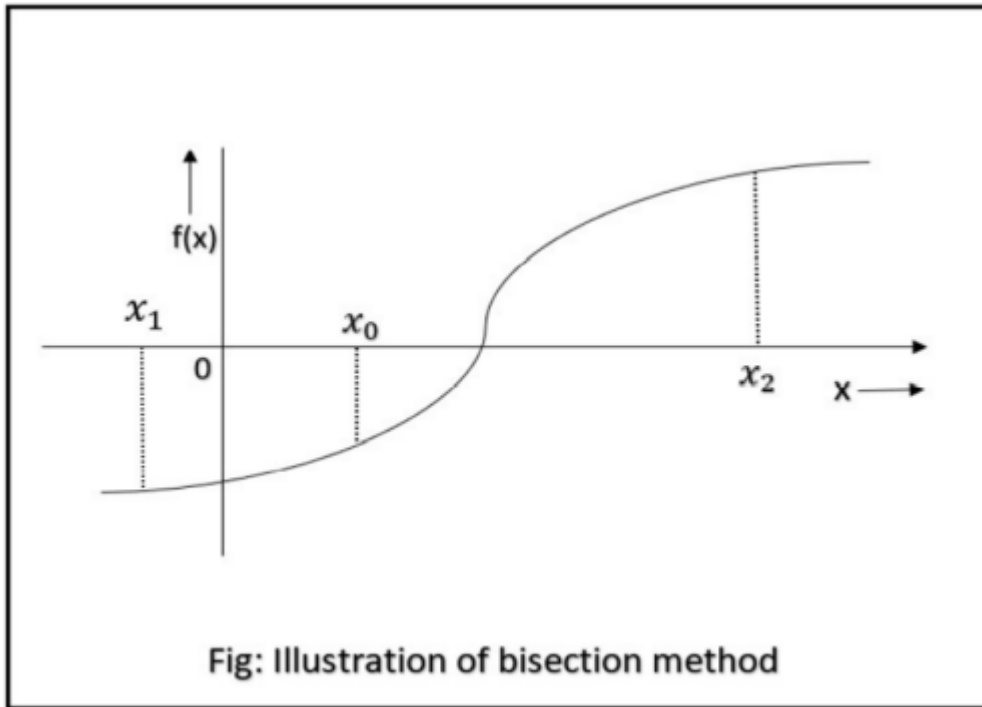
BISECTION METHOD ALGORITHM

To acquire the solution for the continuous function, follow the steps below:

If f is a continuous function, then (x),

Find two points, say a and b, where a b and f(a)* f(b) 0 are true.

If f(t) = 0, the midpoint of a and b is "t," and it is the root of the given function; otherwise, proceed to the following step.

Divide the interval [a, b] – If f(t)*f(a) = 0, a root exists between t and a; otherwise, if f(t) *f (b) = 0, a root exists between t and b.

Repeat the first three stages until f(t) equals 0.

The bisection method is a method of approximating the roots of a given equation by dividing the interval repeatedly. This procedure divides the interval until the resulting interval, which is exceedingly narrow, is discovered.

BISECTION METHOD EXAMPLE

Question: Determine the root of the given equation $x^2-3 = 0$ for $x \in [1, 2]$

Solution:

Given: $x^2-3 = 0$

Let $f(x) = x^2-3$

Now, find the value of f(x) at a= 1 and b=2.

$f(x=1) = 1^2-3 = 1 – 3 = -2 < 0$

$f(x=2) = 2^2-3 = 4 – 3 = 1 > 0$

The given function is continuous, and the root lies in the interval [1, 2].

Let "t" be the midpoint of the interval.

I.e., t = (1+2)/2

t =3 / 2

t = 1.5

Therefore, the value of the function at "t" is

f(t) = f(1.5) = $(1.5)^2$-3 = 2.25 – 3 = -0.75 < 0

If f(t)<0, assume a = t.

and

If f(t)>0, assume b = t.

f(t) is negative, so a is replaced with t = 1.5 for the next iterations.

The iterations for the given functions are:

Iterations	a	b	t	f(a)	f(b)	f(t)
1	1	2	1.5	-2	1	-0.75
2	1.5	2	1.75	-0.75	1	0.062
3	1.5	1.75	1.625	-0.75	0.0625	-0.359
4	1.625	1.75	1.6875	-0.3594	0.0625	-0.1523
5	1.6875	1.75	1.7188	-01523	0.0625	-0.0457
6	1.7188	1.75	1.7344	-0.0457	0.0625	0.0081
7	1.7188	1.7344	1.7266	-0.0457	0.0081	-0.0189

CONCLUSION

The safest method is bisection, which always converges. For a continuous function, the bisection approach is the simplest of all other methods and is guaranteed to converge. Finding the number of steps required for a given precision is always achievable. Bisection method can also be used to design new methods, and bisection method is particularly important in computer science research.

REFERENCES

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[4] Example Courtesy : Byju's